Analysis of Light Guiding Property in Light Piped Based Solar Concentrator

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Abstract-Recently, many researchers have tried to design a system for indoor illumination because the benefits of solar systems. A simple parabolic reflector is often used to collect sunlight but the efficiency is poor when sunlight isn't incident normally. Therefore, an accurate machine to track sun has to be used. In order to get better tolerance, a light pipe based solar concentrator (LPBSC) which comprises a parabolic reflector and a hollow reflective light pipe is proposed. We develop a math model which combines the reflection times of sunlight in light pipe and the candela data of parabolic reflector to analyze the efficiency. And then, straight light pipe is replaced by tapered light pipe to improve the tolerance. Optical simulation software, TracePro, and mathematical software, MATLAB, are used to prove the model is correct and feasible. In the results, LPBSC can improve the tolerance to get good efficiency.

1 INTRODUCTION

Solar systems have many benefits, including energy savings, good quality and healthy illumination to improve worker productivity.[1] Sunlight is the best light source because its color rendering index is 100. Therefore, many researchers have tried to accomplish sunlight as the indoor illumination. General sunlight illumination system can be divided into two systems.[2] One is shading system and the other is optical system. The design of shading system is to block the direct sunlight and guide the diffuse or reflective light. The design of optical system is to redirect sunlight by some optical elements, such as light pipe, lens, etc.

Optical system can be divided into three parts in detail. First part is the solar concentrator. The important factor of this design is the efficiency of collecting sunlight. Second part is the guiding light device. If a nice guiding device is used, light will transport longer without much consumption. Third part is the illumination system. The goal of this design is to achieve uniform maps for indoor illumination. Many researches use parabolic reflector to collect sunlight because the parallel beams entering the reflector will converge at its focal point.[3-4] However, this system has an obvious problem. It needs an accurate machine to track the sun. If the machine isn't accuracy, sunlight won't incident the reflector normally and the collection of sunlight will have a departure from the focal point.

Light pipe plays an important role in sunlight illumination system. Light pipe is a device that can guide light from input to output,[5-7] and it is widely used for projectors,[8] headlights, automotive interior illumination systems,[9] etc. The utility of light pipe is to transport rays and mix rays to get a uniform map, such as European-Commission's project, ARTHELIO.[10] In this paper, we use math model of LPBSC to design a solar system and improve the tolerance and efficiency.

The organizations are as follows. Section 2 includes four parts. First, a typical configuration of LPBSC is introduced. Second part is the analytic derivations of rays in light pipe. Third part introduces how to get candela data in this system. Fourth, math model of LPBSC is found to calculate the efficiency. Finally, an equation is used to analyze the tolerance. Section 3 includes the proof of math model, analysis of the efficiency and tolerance. The conclusion is offered in section 4.

2 METHOD FOR SOLAR CONCENTRATOR DESIGNS

2.1 A typical solar concentrator

A typical configuration of light pipe based solar concentrator (LPBSC) is shown in Fig 1. It consists of a parabolic

Integrated Optics: Devices, Materials, and Technologies XII, edited by Christoph M. Greiner, Christoph A. Waechter
Proc. of SPIE Vol. 6896, 68961A, (2008) · 0277-786X/08/$18 · doi: 10.1117/12.760664

reflector and a hollow reflective light pipe. Because of the long distance from the sun to the Earth, sunlight is parallel. The parallel beams entering the parabolic reflector will converge at its focal point. We dig a hole on light pipe and the hole of light pipe is placed at the focal point of the parabolic reflector. The parabolic reflector transforms sunlight into its focal point. And then, light pipe guides sunlight to two sides. Finally, the collection of sunlight will as the indoor illumination to save energy resources.

The major efficiency consumption of LPBSC is the reflection times of light in light pipe. In follow three sections, analysis of light transporting in light pipe is used to find the reflection times and candela data is a correction factor to correct the intensity in different angles. Finally, a math model to combine reflection times and candela data is used to calculate the efficiency of LPBSC.

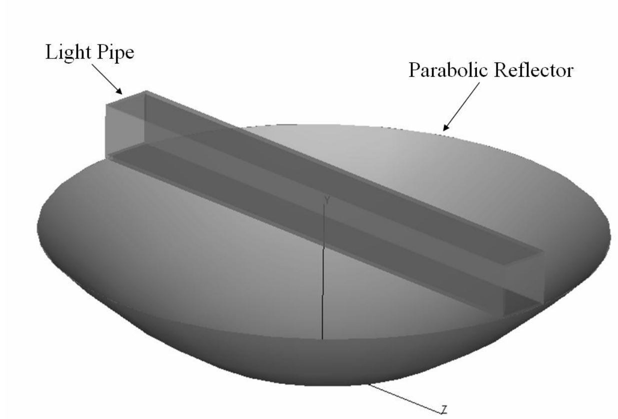


Fig 1 A typical configuration of light pipe based solar concentrator (LPBSC). It comprises a parabolic reflector and a hollow reflective straight light pipe.

2.2 Analysis in Two-Dimensional Light Pipe

The situation of light propagates in light pipe with variation of length, width and extend angle is discussed. In the beginning, consider a tapered light pipe with an input width y_1, an output width a, a length l, and an extend angle δ, as shown in Fig 2. When one ray with an angle θ emits from a point source, the ray will be reflected as it touches the wall of light pipe. The reflection times can divide into two situations. One is even and the other is odd. First, the even reflection times are analyzed in tapered light pipe, as shown in Fig 2(a). In this figure, θ, δ and y_1 are the known numbers, and x_1, x_2 and y_2 are the unknown numbers. In order to get three unknown numbers, three equations bases on simple geometric mathematics have to be given by

$$\begin{cases} x_1 \tan\theta = y_1 + y_2 \\ y_2 = x_1 \tan\delta \\ x_2 = (y_1 + y_2)\cot(\theta - 2\delta) \end{cases} . \tag{1}$$

From above equations, y_2 becomes the known number. Substituting y_2 for $x_1\tan\delta$, x_1 is given by

$$x_1 = \frac{y_1}{\tan\theta - \tan\delta}, \tag{2}$$

and x_2 is given by

$$x_2 = \frac{y_1 \times \tan\theta}{\tan\theta - \tan\delta} \times \cot(\theta - 2\delta). \tag{3}$$

x_1 means the horizontal distance where one ray with an angle θ goes from the bottom to the upper wall of light pipe, and x_2 means the horizontal distance where one ray goes from the upper to the bottom wall of light pipe. The horizontal distance where one ray goes from the bottom and returns to the bottom again is seen in one group. After one group, θ, δ and y_1 are also the known numbers but θ is substituted for θ-2δ. x_0 means the horizontal distance of a ray goes before and it is a correct variable to make the summary equation is accuracy. For example, if it is the first time to calculate (r=1), x_0 is zero. We can summarize the equations and get the reflection times. The summary equation is given by

$$l_{total} = \sum_{n=1}^{r}\{\frac{y_1 + x_0}{\tan[\theta - 2(n-1)\delta] - \tan\delta}\} + \sum_{n=1}^{r}\frac{(y_1 + x_0)\tan[\theta - 2(n-1)\delta]}{\tan[\theta - 2(n-1)\delta] - \tan\delta}\cot(\theta - 2\times n\times\delta) \tag{4}$$

where l_{total} means the horizontal distance where one ray goes from the origin to the last reflection, r means the group number. By equation 4, the reflection times is given by

$$R = 2r\,. \tag{5}$$

Besides the reflection times, the ray's position and angle in the output are found. The equations are given by

$$y' = [l - l_{total}]\tan(\theta - 2\times R\times\delta) \tag{6}$$

and

$$\theta' = \theta - 2\times R\times\delta\,, \tag{7}$$

where y' means the position where one ray ends in y-axis, and θ' means that the ray angle in the output. It is familiar to analyze the odd reflection times in tapered light pipe, but the equation is a little different. The reflection times minus one time as compared with even reflection times. The equation is given by

$$l_{total} = \sum_{n=1}^{r}\{\frac{y_1 + x_0}{\tan[\theta - 2(n-1)\delta] - \tan\delta}\} + \sum_{n=1}^{r-1}\frac{(y_1 + x_0)\tan[\theta - 2(n-1)\delta]}{\tan[\theta - 2(n-1)\delta] - \tan\delta}\cot(\theta - 2\times n\times\delta)\,. \tag{8}$$

The reflection times is given by

$$R = 2r - 1 \tag{9}$$

We also find the ray's position and angle in the output, but it can divide into two situations. One is shown in Fig 2(b) and the other is shown in Fig 2(c). In Fig 2(b), the ray's position and angle equations are given by

$$\theta' = 2\times R\times\delta - \theta \tag{10}$$

and

$$y' = l_{total}\tan\theta - [l - l_{total}]\tan(\theta - 2\times R\times\delta) \tag{11}$$

In Fig 2(c), the ray's position and angle equations are given by
Equation 11 and equation 7.

If the straight light pipe is used, as shown in Fig 3, the equation to find reflection times will be simpler. δ and x_0 is substituted for 0 in equation 4 or equation 8. A simple equation to find reflection times in straight light pipe is given by

$$R = INT\{\frac{L - (a-y)\cot\theta}{a\times\cot\theta} + 1\}\,, \tag{12}$$

where R means the reflection times and INT means the abbreviation of integer. By those equations, we can trace light propagation in light pipe and find any data of light.

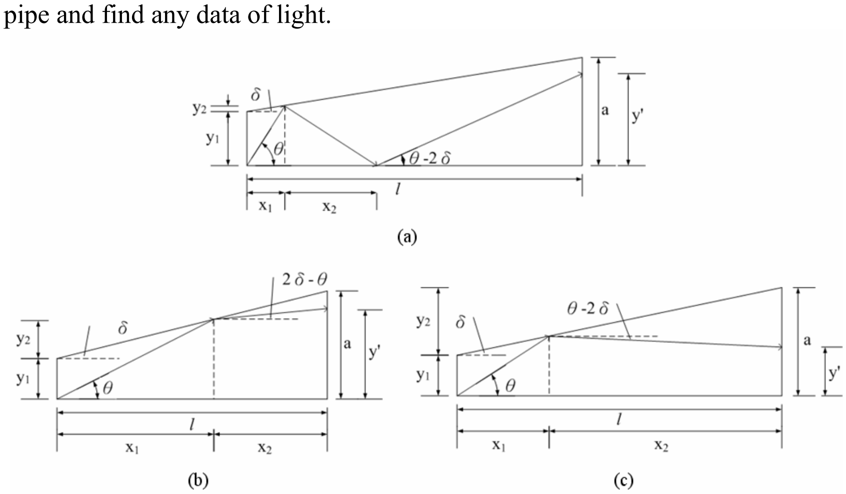


Fig 2 Ray transports in tapered light pipe: (a) even case, (b) odd case1, and (c) odd case2.

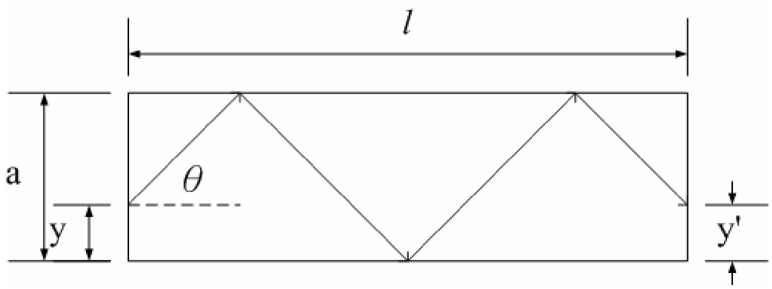


Fig 3 Ray transports in straight light pipe.

2.3 Candela data in mathematics

The reflection times in straight light pipe or tapered light pipe are found by above equations. However, the intensity of ray in different angle isn't the same. We have to find the candela plots at the focal point of the parabolic reflector to correct the efficiency, as shown in Fig 4. A dashed line of Fig 4 is like the shape of the parabolic reflector by theory and a solid line of Fig 4 is the simulation results by TracePro. We want to get candela data without using optical simulation software. We have four steps to accomplish it. An example is shown in Fig 4. First, the equation for the parabolic is given by

$$y = -x^2 + 0.25 \quad (13)$$

Second, we get the coordinates, x and y, by equation 13. Third, the length is given by

$$l = \sqrt{x^2 + y^2}, \quad (14)$$

where l means the length from the sample point to the focal point of parabola. Finally, the candela data is given by

$$W(\theta) = \frac{l(\theta)}{\sum l(\phi)}, \quad (15)$$

where $l(\theta)$ means the length from the sample point to the focal point of parabola versus the different angle and $\Sigma l(\Phi)$ means the summary of length from all the sample points to the focal point of parabola. The candela data is shown in Fig 5. In this figure, we find the weight of any ray to all rays easily.

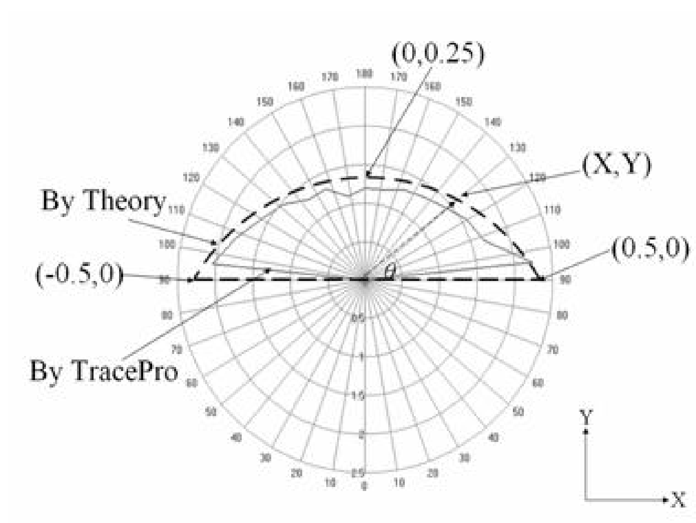


Fig 4 Candela plots at the focal point of the parabolic reflector: dashed line is drawn by theory and solid line is simulated by TracePro.

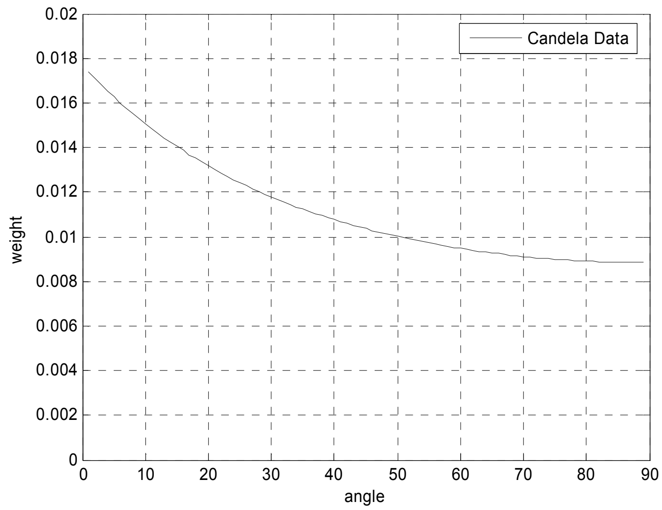


Fig 5 Candela data is calculated by theory.

2.4 The efficiency equation of LPBSC

The efficiency of LPBSC is analyzed by combination of above two sections. One is the reflection times and the other is candela data. The formula to calculate the efficiency versus angle is given by

$$E(\theta) = W(\theta) \times T^{R(\theta)} \tag{16}$$

where $E(\theta)$ means the efficiency versus the angle, $W(\theta)$ means the candela plots at the focal point of the parabolic reflector, T means the reflectance of light pipe, and $R(\theta)$ means the reflection times versus the angle. And then, the efficiency equation is the summation of all rays, as follows

$$E = \sum E(\theta) \tag{17}$$

By equation 17, the efficiency of LPBSC is easy to calculate.

2.5 A modified solar concentrator to improve the tolerance

If the typical configuration of LPBSC is used, the collection of sunlight by parabolic reflector will not incident the hole of straight light pipe when sunlight doesn't incident normally. Therefore, straight light pipe is replaced by tapered light pipe to improve the tolerance. Ramp of tapered light pipe makes the higher angle slower through the reflection. It means that some bottom parts of light pipe don't have rays go through. This system is shown in Fig 6.

And then, an equation is provided to discuss the tolerance, as shown in Fig 8. In this figure, a solid line means sunlight incidents into the reflector normally, and a dashed line means sunlight has a tilt angle. First, we define the sign conventions in this paper, as shown in Fig 7. In this system, the length from the apogee to the focal point is 0.25. The apogee is located on the origin and the focal point is located on the coordinate, (0,0.25). No matter what situation in Fig 8, the departure from the focal point is given by

$$\Delta z = (0.25 - y) \times \tan(2\theta - \delta) - (0.25 - y) \times \tan 2\theta\,, \tag{18}$$

where Δz means that the departure from the focal point when sunlight has an acclivitous angle, y means that the coordinate of the Y-coordinate, θ means the angle between the normal incidence ray and the normal, and δ means the angle between the non-normal incidence ray and the normal. Therefore, we can use this equation to find the departure from the focal point when sunlight has a tilt angle and discuss the tolerance.

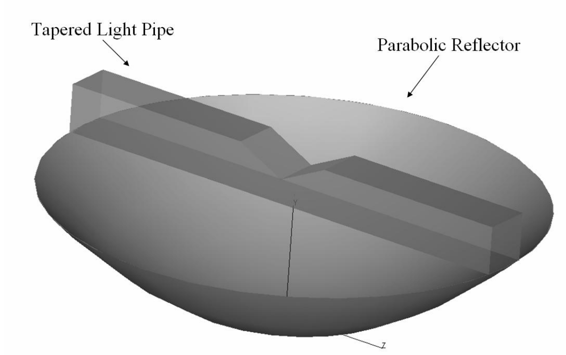


Fig 6 A modified configuration of light pipe based solar concentrator (LPBSC). It comprises of a parabolic reflector and a hollow reflective tapered light pipe.

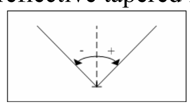


Fig 7 Sign conventions as used in the paper.

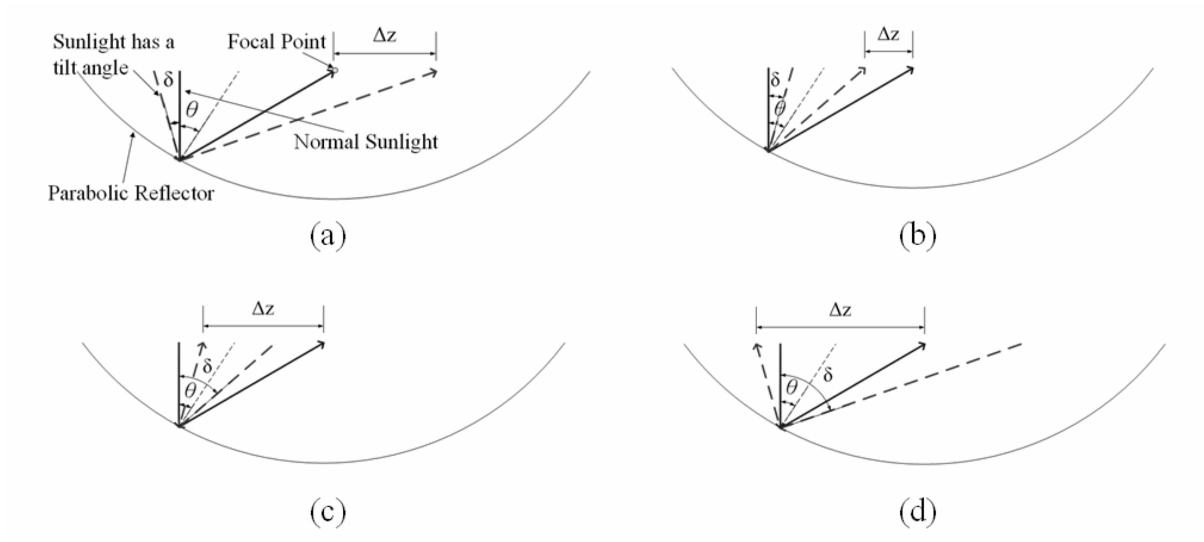


Fig 8 When sunlight has a tilt angle, the departure from the focal point. The solid line means sunlight incidents normally and the dashed line means sunlight has a tilt angle. There are four situations: (a)δ and θ are the different sign convention (b)δ and θ are the same sign convention and δ<θ (c)δ and θ are the same sign convention and δ>θ (d)δ and θ are the same sign convention and δ>2θ.

3 RESULT AND DISCUSS

3.1 Prove the efficiency equation

We use TracePro and MATLAB to prove that the efficiency equation is accuracy. By above equations and TracePro, the efficiency versus different widths of the light pipe is calculated individually, where the widths of the light pipe are 0.01 to 0.1 with an interval of 0.01. The results are shown in Fig 9. In this figure, the efficiency calculates by equations is always greater than by TracePro. The reason for this situation is the candela plots. We find out the dashed line and solid line of Fig 4 is a little different near 0 degree and 180 degree. The dashed line is a smooth plot from 0 degree to 180 degree but the solid line has no ray near 0 degree and 180 degree. For this reason, the efficiency calculates by equations is always a little greater than by TracePro, but the errors between equations and TracePro are only about one percent. Therefore, we can use the equations we provided as the first order to design LPBSC.

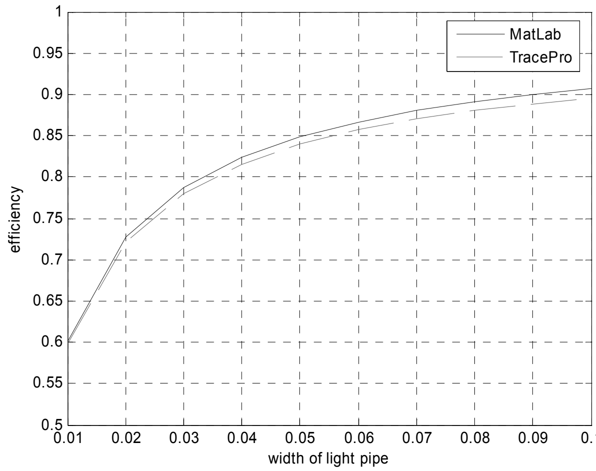


Fig 9 The efficiency versus different widths are simulated by MATLAB and TracePro individually to find the error between theory and simulation.

3.2 The tolerance of modified solar concentrator

Using modified solar concentrator can improve the tolerance, because tapered light pipe makes the higher ray angle lower. It means that somewhere in the bottom of the tapered light pipe won't have any rays. Therefore, the hole in tapered light pipe will be made bigger, and it doesn't affect the efficiency of LPBSC. The collection of sunlight by the parabolic reflector is still guided in light pipe, when sunlight is tilted an angle. If a parabolic reflector is only used, it will have no ray to incident at the focal point and the efficiency will be poor.

In this section, the departure for tilt angle which -5 to 5 degree with an interval of one is simulated. The results are shown in Fig 10. The x-axis describes the x coordinate sun light incidents on the reflector. The y-axis describes the departure from the focal point. We find the closer to zero one ray is, the bigger the departure from the focal point. If the departure below 0.1 is accepted, the ratio of incident rays to all rays is shown in Fig 11. Seventeen percent of all rays can incident to light pipe when sunlight tilt angle is -10 or 10 degree. Therefore, it is feasible to improve the tolerance by changing the shape of light pipe.

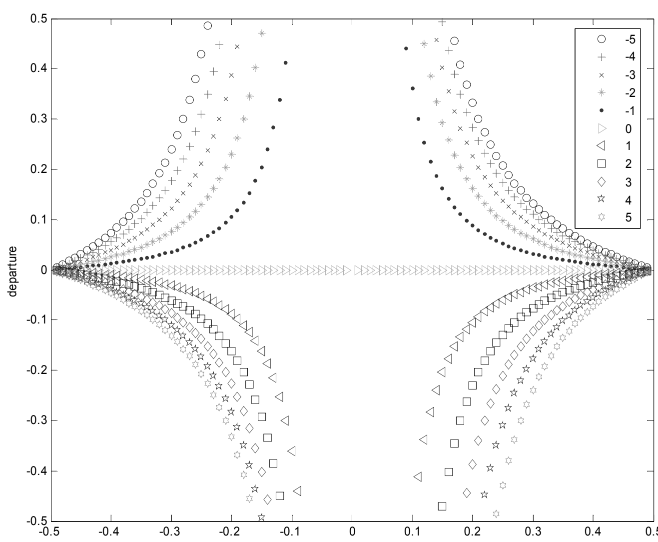


Fig 10 When sunlight has a tilt angle,-5 to 5 degree, the ray departure from the focal point after reflecting by the parabolic reflector.

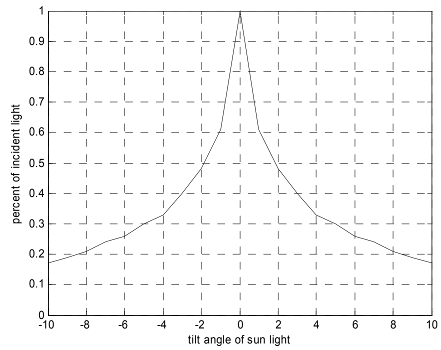


Fig 11 If the departure below 0.1 is accepted, this figure is the ratio of incident rays to all rays for the tilt angle from -10 to 10 degree.

3.3 The efficiency of typical and modified solar concentrator

The efficiency of typical and modified configuration of LPBSC is discussed. First, the figure of efficiency versus angles with three different widths is shown in Fig 12. In this figure, we find that the lower angle the ray starts, the better the efficiency will be, no matter what width is. Because of more reflection times, $T^{R(\theta)}$ of equation 16 will be smaller. For this reason, the efficiency of the higher reflection times will be smaller than the lower reflection times. We will try to make the ray of higher angle more efficiency. The straight light pipe will be replaced by the tapered light pipe. The ramp of the tapered light pipe makes the higher ray angle lower.

The modified solar concentrator comprises one parabolic reflector and one hollow light pipe. The light pipe is made of one tapered light pipe and one straight light pipe, like two systems. In order to get the efficiency of the modified solar concentrator, first, the candela data have to be gotten. Then we use equation 4 to equation 11 to calculate the reflection times, positions and angles in the output of tapered light pipe. The positions and angles in the output will be the new source data in the next system. Then the reflection times in straight light pipe is calculated by equation 12. Now, the candela data and the reflection times are known. Finally, we use equation 16 and equation 17 to calculate the efficiency of modified system. Besides, because we have all equations, we can fix some variables to optimize this system. For example, the width of the light pipe is 0.1 and the efficiency of typical and modified configuration is calculated individually. The results are shown in Fig 13. We find that the efficiency in 60 to 90 degree is improved in modified system clearly.

Optical simulation software, TracePro, is used to simulate the typical system and modified system to find irradiance maps to analyze the efficiency and uniformity. In our simulations, sunlight is set 1 watt and the parameters of two light pipes in our simulation are displayed in Table 1. The results are shown in Fig 14 and Fig 15. The efficiency is 0.70463 watt in the typical system and the efficiency is 0.76946 watt. In a word, using modified LPBSC is not only to improve the tolerance but also to raise the efficiency.

Table 1 Properties of light pipe in typical and modified system.

Name	Parameters	Values
Straight light pipe	Width	0.1
	Length	0.5
	Reflectance	0.99
Tapered light pipe	Input width	0.055698
	Output width	0.1
	Length	0.5
	Extend angle	37.5
	Reflectance	0.99

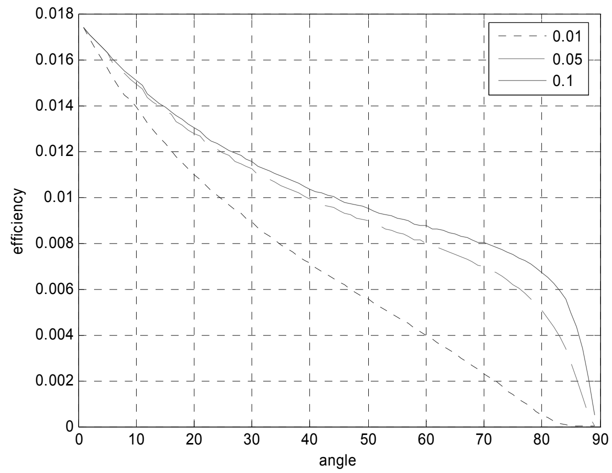


Fig 12 The efficiency of every angle for three different widths are calculated by MATLAB. Three widths of light pipe are calculated. One is 0.01 (dotted line), another is 0.05 (dashed line) and the other is 0.1 (solid line).

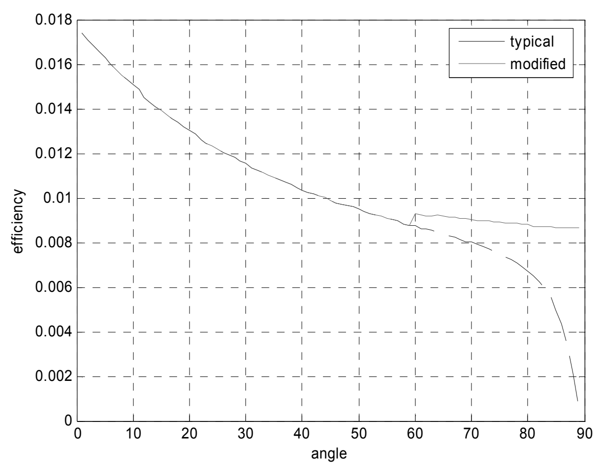
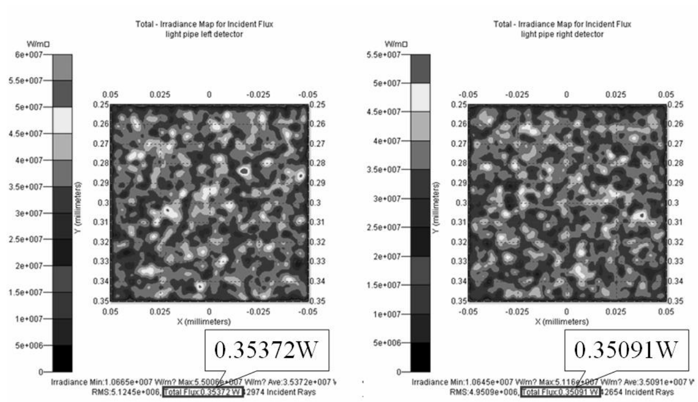


Fig 13 The efficiency of every angle for two configurations are calculated by MATLAB. One is using straight light pipe (typical) and the other is using tapered light pipe (modified).



(a) (b)

Fig 14 Irradiance maps of typical configuration: The detectors are (a) left side of light pipe (b) right side of light pipe

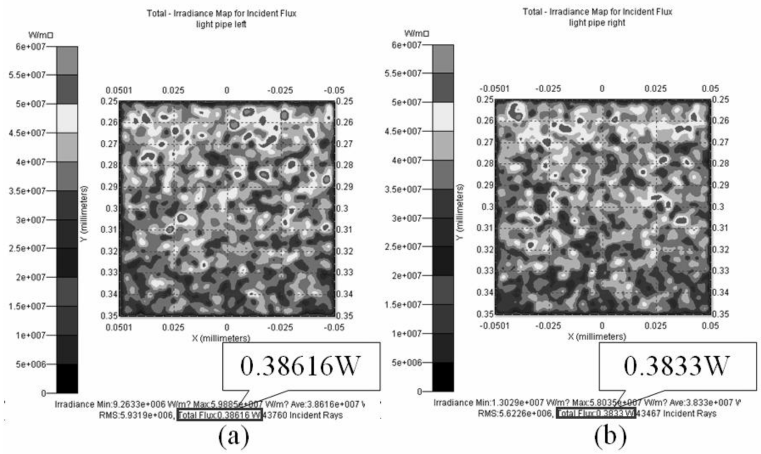


Fig 15 Irradiance maps of modified configuration: The detectors are (a) left side of light pipe (b) right side of light pipe

4 CONCULSION

Although the speed of today's computers is faster than before, it's also consuming to construct CAD and trace rays. Math model of LPBSC we provided are used to analyze and design this system as a first-order design tool. By above equations, the shape of light pipe or the size of parabolic reflector is easy to change and the efficiency is calculated in a second.

A construction of solar concentrator is provided. The prototype of LPBSC is designed by math model. We found that when the shape of light pipe is changed, the benefits included the improved tolerance and efficiency. The construction using straight light pipe and tapered light pipe is simulated individually by TracePro. The efficiency of typical configuration is 70.463 percent, and the efficiency of modified configuration is 76.946 percent. The modified configuration is more efficient than typical configuration about 6.483 percent.

The tolerance of LPBSC is also analyzed. We find how many percent of light can incident to the light pipe when sunlight has a tilt angle. In the results, even when sunlight has a tilt angle 10 degree or -10 degree, 17 percent of light can incident to the light pipe. It's better than only using parabolic reflector to collect sunlight.

Future work of this paper will emphasize the optimization of light pipe by math model. The goal is to find an adaptable shape of light pipe to get better tolerance and efficiency by math model.

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